# The determination of drag in front crawl swimming 

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Accepted 17 February 2004


#### Abstract

The measurement of drag while swimming (i.e. active drag) is a controversial issue. Therefore, in a group of six elite swimmers two active drag measurement methods were compared to assess whether both measure the same retarding force during swimming. In method 1 push-off forces are measured directly using the system to measure active drag (MAD-system). In method 2 (the velocity perturbation method, VPM) drag is estimated from the difference in swimming speed when subjects swim twice at maximal effort (assuming equal power output and assuming a quadratic drag-speed relationship): once swimming free, and once swimming with a hydrodynamic body attached that created a known additional resistance. The average drag for the VPM tests ( 53.2 N ) was statistically significant and different from the active drag for the MAD-test $(66.9 \mathrm{~N})$, paired Student's $t$-test: 2.484, $12 \mathrm{DF}, p=0.029$. A post hoc analysis was performed to assess whether the two methods measure a different phenomenon. Based on the drag speed curve obtained with the MAD-system, the VPM-data were re-examined. For diverging drag determinations the assumption of equal power output of the 'free' trial (swimming free) vs. the towing trial (swimming with hydrodynamic buoy) appeared to be violated. The regression of the relative difference in force (MAD vs. VPM) on the relative difference in power (swimming free vs. swimming with hydrodynamic body) was: $\% \Delta \mathrm{drag}=1.898 \times \% \Delta$ power -4.498 , $r^{2}=0.88$. This suggests that the major part of the difference in active drag values is due to a non-equal power output in the 'free' relative towing trial during the VPM-test. The simulation of the violation of the equal power output assumption and the calculation of the effect of an other than quadratic drag-speed relationship corroborated the tentative conclusion that both methods measure essentially the same phenomenon and that active drag differences can be explained by a violation of test assumptions.


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Keywords: Swimming; Front crawl; Active drag; Velocity perturbation method; MAD-system

## 1. Introduction

Human swimming performance is poor when compared to species whose habitat is aquatic. A maximum swimming speed of approximately $2 \mathrm{~m} \mathrm{~s}^{-1}$ represents only about $16 \%$ of the maximum unaided speed attained on land. One obvious reason for this speed difference is the higher resistance one encounters when

[^0]moving through water. It is therefore not surprising that throughout the history of swimming research, attempts have been made to determine this resistance. As early as 1905, Dubois-Reymond (1905) towed people behind a rowing boat, measuring resistance with a dynamometer. Liljestrand and Stenstrom (1919) measured resistance towing swimmers by means of a windlass on shore. Amar (1920) was the first to assume that the resistance is related to the square of the swimming speed
$D=K v^{2}$
in which $D$ denotes drag, $K$ is a constant, and $v$ the swimming speed. Karpovich (1933) used a 'natograph' to register drag dependence on speed. Both Amar (1920) and Karpovich (1933) used measurement techniques
determining the resistance of swimmers gliding passively through the water. The relation between resistance $(N)$ and speed $\left(\mathrm{m} \mathrm{s}^{-1}\right)$ based on their experiments was approximately $D=29 v^{2}$. However, the body is never in a stable prone position when swimming, since some propulsive forces need to be generated. It was conjectured that the movements necessary to create propulsion could induce additional resistance. This resulted in attempts to determine the drag of a person who is actively swimming. Techniques to determine this active drag were developed by several groups in the 1970s (Clarys and Jiskoot, 1975; Clarys et al., 1974; Holmér, 1974; di Prampero et al., 1974; Rennie et al., 1975). A common feature of the methods developed was their adoption of extrapolation techniques. In the method used by Holmér (1974), di Prampero et al. (1974), and Rennie et al. (1975), the variation in oxygen consumption as a result of small extra forces applied to the swimmer is extrapolated. In the method introduced by Clarys et al. (1974), variations in external forces applied on a moving carriage as a function of imposed speed variations are extrapolated. Both methods yielded comparable results and, as expected, higher values $(150-300 \%)$ than the previously reported values for passive drag.

In the mid-1980s, Hollander et al. (1986) developed an approach to measure active drag (MAD-system). The technique relies on the direct measurement of the pushoff forces while swimming the front crawl. Later on, Kolmogorov and Duplisheva designed yet another method to determine the active drag (Kolmogorov and Duplisheva, 1992). In their so-called velocity perturbation method (VPM) or method of small perturbations, subjects swim a 30 m lap twice at maximal effort: once swimming 'free', and once swimming while towing a hydrodynamic body that creates a known additional resistance. For both trials, the average speed is calculated. Under the assumption that in both swims the power output to overcome drag is maximal and constant, active drag can be calculated since power to drag equals drag force times speed:
$D_{\mathrm{f}} v_{\mathrm{f}}=D_{\mathrm{t}} v_{\mathrm{t}}$,
where the subscripts refer to the swims in the 'free' and towing trials. Using Eq. (1), this can be cast into
$K v_{\mathrm{f}}^{3}=K v_{\mathrm{t}}^{3}+F_{\mathrm{b}} v_{\mathrm{t}}$,
where $F_{\mathrm{b}}$ represents the added drag due to the hydrodynamic body. Since the hydrodynamic properties of this added body were calibrated previously, it was possible to compute $F_{\mathrm{b}}$ at any speed. Then, $K$ can be solved and since $D_{\mathrm{f}}=K v_{\mathrm{f}}^{2}, D_{\mathrm{f}}$ will equal:
$D_{\mathrm{f}}=K v_{\mathrm{f}}^{2}=\frac{F_{\mathrm{b}} v_{\mathrm{t}} v_{\mathrm{f}}^{2}}{v_{\mathrm{f}}^{3}-v_{\mathrm{t}}^{3}}$.

The interesting aspect of this approach is that it can be applied to measure active drag in all four competitive strokes, while the MAD-system and indirect methods are applicable only to the front crawl. However, the approach will yield only one drag estimate at maximal speed.

When these more recent techniques were used to estimate active drag (Kolmogorov and Duplisheva, 1992, Kolmogorov et al., 1997; Toussaint et al., 1988b; van der Vaart et al., 1987) considerably lower values were found than active drag values reported in the seventies (e.g. Clarys and Jiskoot, 1975). Except for the VPM, the recent active drag values were comparable to values reported earlier for passive drag (i.e. $D=26 v^{2}$ ). Kolmogorovs approach yielded even lower values: $D=16 v^{2}$. The state of affairs regarding measuring active drag in human swimming was summarised by Hay (1988):
> "consistency is not a feature of the results obtained in studies on active drag... One can hardly expect to evaluate a swimmer's ability to minimize resistance... if one cannot even measure the forces involved with some degree of accuracy."

In the present study, the results of two methods (MAD-system and VPM) to measure active drag are compared with each other. In the MAD-system approach, the swimmer's technique is altered since the push off is made from fixed push off pads rather than from moving water. So it can be questioned whether this could affect drag characteristics. In the VPM, swimmers are assumed to deliver equal power to overcome drag in the 'free' and the towing trial. Also, it is assumed that drag relates to speed squared, which might not be completely correct at higher swimming speeds where wave formation becomes more important (Toussaint et al., 2002). A more pronounced wave formation could lead to a higher than quadratic increase of drag with speed. The prime aim of this study is to determine whether the MAD-system and the VPM are measuring the same phenomenon (i.e. active drag). Hence, if results differ, the question is whether this is due to a violation of test-assumptions or whether the two methods measure an essentially different phenomenon. Therefore a post hoc analysis will be used to examine two test assumptions:
(I) the assumption of equal power output: using the drag speed curve obtained with the MAD-system and the drag characteristic of the hydrodynamic buoy, the power output in the 'free' trial is compared to that in the towing trial of the VPM test, given the speeds recorded in both trials. If the power output in the 'free' trial is different from the towing trial, to what extend will this influence the VPM-drag estimate?
(II) the assumption of a quadratic drag-speed relationship: assuming equal power output in the 'free'
and towing trial, the effect of a deviation of the exponent from 2 on the calculation of the VPM-drag is assessed.

## 2. Methods

Six top-level international competitive swimmers from the swimming team TZA (Top Zwemmen Amsterdam, see Table 1 for details) participated in this study after a written informed consent was obtained. The drag-speed relationship of each swimmer was determined using the MAD-system. For the same subjects, drag was determined employing the VPM. Measurements were made on several days within a period of 6 weeks. For swimmers C, S, and T it was possible to do three tests with the VPM. Other swimmers failed to participate in one (R) or two (B and E) tests due to school obligations, training camp and/or illness.

### 2.1. Measurement of active drag (MAD system)

The MAD-system (Toussaint et al., 1988a) enables the swimmer to push off from fixed pads at each stroke. These push-off forces are measured. The swimmer is instructed to swim at a constant speed. At constant swimming speed, the mean propelling force is equal to the mean drag force (van der Vaart et al., 1987).

The push-off pads are attached to a 23 m long rod. The distance between the push-off pads was 1.35 m , while the rod was mounted 0.8 m below the water surface. The rod was instrumented with a force transducer in order to measure the push-off forces. The force signal was low pass filtered $(15 \mathrm{~Hz}$ cut-off frequency), digitised ( 100 Hz sampling frequency), processed and stored on disk using an Apple PowerBook ${ }^{\circledR}$ G4. The force signal from the second push-off to the last (16th pad) is time-integrated and yields the average force

Table 1
Individual Data for Gender, Age, Mass, Height, and short course front crawl performance

| Subject | Age <br> (year) | Gender | Height <br> $(\mathrm{m})$ | Mass <br> $(\mathrm{kg})$ | 100 m <br> time (s) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| B | 27 | M | 1.97 | 75 | 50.5 |
| C | 19 | F | 1.75 | 62 | 55.1 |
| E | 23 | M | 2.00 | 83 | 49.4 |
| R | 17 | M | 1.85 | 74 | 51.8 |
| S | 19 | M | 1.84 | 72 | 51.1 |
| T | 19 | M | 1.86 | 72 | 53.8 |
|  |  |  |  |  |  |
| Mean | 20.7 |  | 1.878 | 73.0 | 51.95 |
| SD | 3.7 |  | 0.092 | 6.75 | 2.13 |

while swimming $(14 \times 1.35 \mathrm{~m}) 18.9 \mathrm{~m}$. The mean speed was computed from the time taken to cover this 18.9 m distance between the second and last pad. The subjects used their arms only; the legs were supported by a small pull buoy. The same buoy was also used when drag was determined using the VPM.

To establish the relationship between drag and swimming speed, subjects were asked to swim 16 lengths $(25 \mathrm{~m})$, each at a different but constant speed (range $1.0-2.1 \mathrm{~m} \mathrm{~s}^{-1}$ ). For each length, mean drag force and mean swimming speed is measured. These 16 speed/drag data are least-squares fitted to the function:
$D=A v^{n}$,
where $D$ represents total active drag, $v$ equals swimming speed and $A$ and $n$ are parameters of the power function. This function is applicable because total drag is dominated by pressure drag at the prevailing high Reynolds's numbers of $2 \times 10^{6}-4 \times 10^{6}$ (Toussaint et al., 1988b). For each subject, $A$ and $n$ are obtained via a least square fit with a Levenberg-Marquardt algorithm using Matlab (Mathworks, Natick, MA). These fitted functions are used to estimate the drag at the speed for which drag is estimated using the VPM.

### 2.2. Velocity perturbation method (VPM)

Active drag of the swimmer is measured by the method of small velocity perturbations also known as method of small perturbation (Kolmogorov and Duplisheva, 1992; Kolmogorov et al., 1997) that was carried out in a 50 m pool. The subjects are asked to swim a 25 m distance twice at maximal effort swimming arms only with the pull buoy supporting their legs: once swimming 'free', and once towing a hydrodynamic body that created additional resistance. The hydrodynamic body has a drag characteristic of $F_{\mathrm{b}}=9.32 v^{2}$ and is attached with an 8 m long rope to a belt around the waist of the swimmer. The approximately 8 m distance between swimmer and hydrodynamic body ensured that the drag force of the buoy was unaffected by the wake created by the swimmer (Kolmogorov and Duplisheva, 1992).

For both trials, the time to cover the 25 m was registered with the use of two coupled video cameras ( 50 fields/s) with synchronised time code registration. The cameras were positioned 25 m apart with the optical axis perpendicular to the path of the swimmer. The start and finish line of the 25 m were marked with vertical bars positioned on the side of the pool that were clearly visible in the video recordings. The time difference $(\Delta t)$ between the head passing the start line on the first camera and the finish line on the second camera was calculated. The accuracy of this method is 0.02 s . The average swimming speed of each trial was calculated
with this time difference according to
Speed $=25 / \Delta t$.
Under the assumptions that in both swims an equal amount of power is available to overcome drag, and drag relates to speed squared, active drag can be calculated using Eqs. (2)-(4). Hence, it is assumed that propelling efficiency (Toussaint et al., 1988a) is equal to swimming 'free' and swimming with the hydrodynamic body attached.

### 2.3. Post Hoc analysis of the equal power output assumption

The drag-speed relationship for the swimmer (MADresults) and the drag characteristic of the hydrodynamic buoy, enable the calculation of power to overcome drag in the 'free' $\left(P_{\mathrm{df}}\right)$ and towing trial $\left(P_{\mathrm{dt}}\right)$ as calculated based on the measured speeds according to
$P_{\mathrm{df}}=A v_{\mathrm{f}}^{n+1}$ and $P_{\mathrm{dt}}=A v_{\mathrm{t}}^{n+1}+F_{\mathrm{b}} v_{\mathrm{t}}$.
The effect of a difference in power to drag between the 'free' and towing trial on the difference between MAD-drag and VPM-drag was simulated in Matlab (see the Appendix A for program listing). Two approaches to calculate the effect of a difference in power output between the 'free' and towing trial on the VPM-drag estimate are applied; (I) using the MAD-drag curve (i.e. Eq. (5)) and (II) using the VPM results (i.e. Eq. (4)). The effect of a difference in power output in the towing trial was calculated for power values between $65 \%$ and $105 \%$ of the power in the 'free' trial. Both calculations (I and II) provide an indication as to how sensitive the VPM-drag estimate is for the violation of the equal power output assumption.

### 2.4. Post Hoc analysis of a quadratic drag-speed relationship

A simulation was performed in which the effect of the value of the drag exponent on the calculation of the active drag force using the VPM-approach (i.e. Eq. (4)) was calculated. The exponent was varied between 1.9 and 2.7 in steps of 0.01 (see listing in the Appendix A). The result gives an indication how sensitive the VPMdrag estimate is when the true exponent of the dragspeed relationship deviates from 2.

## 3. Results

Statistically significant different active drag values are found (see Fig. 1 and Table 2): The average drag for the VPM test was 53.2 N , while the active drag for this speed based on the MAD-test was 66.9 N . A $t$-test revealed a statistically significant difference in active drag values:
paired Student's $t$-test: $t$-value of $2.484,12 \mathrm{DF}$, $p=0.029$. However, some of these results (for subjects C, E, and S) match the MAD-results quite well, while other drag recordings are half the value obtained with the MAD-system.

A post hoc analysis revealed that there was a significant difference in power to drag between the 'free' and towing trial $(\Delta P=13.2 \mathrm{~W})$ that can explain for the observed difference in MAD-drag and VPMdrag. The relative power difference correlated well with the relative difference in force $(r=0.94, p<0.001$, see Fig. 2). The intercept of the regression was with -4.5 , not statistically different from zero, suggesting that the difference between MAD-drag and VPM-drag values can be explained by a violation of the equal power output assumption swimming the 'free' trial and the one towing the hydrodynamic buoy.

### 3.1. Sensitivity analysis of the violation of the equal power assumption

The calculation of the effect of a relative difference in power output on the relative difference in MAD-drag and VPM-drag fitted the actually measured data remarkably well (Fig. 3). In this calculation the average drag-speed relationship of all MAD-tests (see Table 2) was used ( $\mathrm{drag}=21.33 v^{2.34}$ ) to estimate the power output to overcome drag given the average speed of all 'free' trials (111 W).

The calculation of the effect of a relative power output difference on relative drag estimates taking the VPM approach as starting point also fitted the actually measured data remarkably well (Fig. 4). In this calculation the data of subject $S$ were used as a starting point since for this subject both methods give quite similar drag values when the average speeds for the 'free' and towing trial are taken (MAD 67.8 N ; VPM 65.5 N , a $3.4 \%$ difference). The power output to drag in the 'free' trial is then 113 W . If power output in the towing trial is different, speed in the towing trial will diverge from the average of $1.53 \mathrm{~m} \mathrm{~s}^{-1}$. This change in speed was used to simulate the effect of changes in power output (see listing in the Appendix A).

### 3.2. Sensitivity analysis of the exponent in the drag speed relationship diverging from 2

The effect of the variation of the exponent on the relative difference in active drag estimates (Fig. 5) is small compared to the effect of a non-equal power output in the 'free' and towing trial. In the calculation it was again assumed that actual drag equals $21.33 v^{2.34}$ and power output to drag in the 'free' and towing trial to be equal to 111 W .


Fig. 1. Drag data dependent on swimming speed for all subjects. Each filled dot represents the speed-drag combination of swimming one lap on the MAD-system. Fitted curves are presented as well. The results for the VPM-test are indicated with a buoy icon.

## 4. Discussion

The active drag measured with the MAD-system is significantly different from that measured with the Velocity Perturbation Method. However, closer inspection of the individual active drag results for the two methods gives a mixed picture; for some of the subjects the results agree well (see results for subject $\mathrm{C}, \mathrm{E}$, and S in Fig. 1), for others the VPM-drag results are half the value obtained with the MAD-system (subjects R and T). Do the latter results confirm the average finding and should we conclude that the two methods measure a different phenomenon? We decided to adopt the hypothesis that both approaches do actually measure the same active drag, but that violations of key assumptions in the

VPM-tests could be responsible for the difference in results.

### 4.1. Analysis of the equal power output assumption

The comparison of the power output in the 'free' and towing trial reveals significant differences indicating a violation of the equal power output assumption. The deviation of power in the towing trial expressed as a percentage of that in the 'free' trial correlates well with the relative deviation in drag estimates (see Fig. 2). Given that the intercept of the regression was not statistically significant from zero, it suggests that the difference in drag results can be explained by the difference in power output in the 'free' and towing trial.

Table 2
Least squares fitted parameters describing the curves of the drag dependent on speed ( $F_{\mathrm{d}}=A v^{n}$ ).

| Subj. | $A$ | $n$ | Free <br> trial <br> speed $\left(\mathrm{m} \mathrm{~s}^{-1}\right)$ | towing trial speed ( $\mathrm{m} \mathrm{s}^{-1}$ ) | Drag <br> VPM <br> (N) | Drag <br> MAD <br> (N) | $\Delta$ drag (N) | $\begin{aligned} & \text { \% } \Delta \text { drag } \\ & (\%) \end{aligned}$ | Power <br> 'free' <br> trial (W) | Power <br> towing <br> trial (W) | $\Delta P(\mathrm{~W})$ | $\begin{aligned} & \% \Delta P \\ & (\%) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | 24.5 | 2.16 | 1.64 | 1.51 | 87.1 | 71.3 | $-15.78$ | -22.1 | 117 | 121 | -5 | -3.9 |
| C | 16.6 | 2.53 | 1.46 | 1.31 | 50.6 | 43.2 | -7.36 | -17.0 | 63 | 64 | 0 | -0.8 |
| C | 16.6 | 2.53 | 1.54 | 1.30 | 33.0 | 49.0 | 16.01 | 32.6 | 75 | 62 | 14 | 18.2 |
| C | 16.6 | 2.53 | 1.54 | 1.32 | 39.0 | 49.0 | 10.06 | 20.5 | 75 | 66 | 9 | 12.2 |
| E | 24.0 | 2.25 | 1.72 | 1.54 | 71.1 | 81.4 | 10.38 | 12.7 | 140 | 133 | 8 | 5.5 |
| R | 15.3 | 2.82 | 1.77 | 1.45 | 35.6 | 75.7 | 40.13 | 53.0 | 134 | 91 | 43 | 32.1 |
| R | 15.3 | 2.82 | 1.77 | 1.43 | 33.0 | 75.7 | 42.71 | 56.4 | 134 | 87 | 47 | 34.8 |
| S | 20.7 | 2.18 | 1.67 | 1.51 | 77.1 | 63.1 | $-14.00$ | -22.2 | 105 | 110 | -5 | -4.3 |
| S | 20.7 | 2.18 | 1.75 | 1.57 | 75.2 | 70.2 | -5.07 | -7.2 | 123 | 124 | -1 | -0.6 |
| S | 20.7 | 2.18 | 1.75 | 1.51 | 51.0 | 70.2 | 19.22 | 27.4 | 123 | 109 | 14 | 11.5 |
| T | 28.7 | 2.08 | 1.56 | 1.32 | 34.8 | 72.3 | 37.55 | 51.9 | 113 | 89 | 24 | 21.3 |
| T | 28.7 | 2.08 | 1.58 | 1.40 | 54.7 | 74.3 | 19.54 | 26.3 | 117 | 107 | 10 | 8.5 |
| T | 28.7 | 2.08 | 1.58 | 1.39 | 49.1 | 74.3 | 25.15 | 33.9 | 117 | 104 | 14 | 11.6 |
| Mean | 21.33 | 2.34 | 1.64 | 1.43 | 53.2 | 66.9 | 13.7 | 18.9 | 111 | 97.3 | 13.2 | 11.2 |

$A=$ coefficient of proportionality, $n=$ power of the speed; the speed in the 'free' trial and the towing trial of the VPM-test; drag measured by the two methods, the power to overcome total drag in the 'free' and towing trial; the relative differences are expressed in percentages relative the value in the 'free' trial.


Fig. 2. Regression of relative difference in drag between the MADand VPM-method on relative difference in power swimming with (towing trial) and without buoy ('free' trial).

### 4.2. Sensitivity analysis of the violation of the equal power assumption

The sensitivity analysis reveals that the VPM-drag results are sensitive for small differences in power output in the 'free' and towing trials. A $15 \%$ difference in power leads to $30 \%$ error in drag. The curve describing the $\%$-error depending on the $\%$-power difference in the 'free' and towing trial fitted the actually measured data remarkably well irrespective of whether the MADapproach (Fig. 3) or the VPM-approach (Fig. 4) was used as a starting point for the calculation. This suggests


Fig. 3. Sensitivity analysis of equal power assumption: simulated relative difference in drag between the MAD- and VPM-method dependent on relative difference in power swimming with (towing trial) and without hydrodynamic buoy ('free' trial) using the MAD-drag curve as reference. Actual data points are given in the same graph.
that indeed a violation of test assumptions in the VPMapproach lead to mixed results for our subjects.

A comparison of Figs. 3 and 4 reveals a slightly different 'goodness of fit'. In Fig. 4 the calculation procedure using the VPM-approach (i.e. Eq. (4)) 'forces' the curve to intercept at the $3.4 \%$ drag-difference between MAD and VPM for subject $S$ for $v=1.72 \mathrm{~m} \mathrm{~s}^{-1}$ (so the fit would have been 'better' if in the calculation the speed for the towing trial was adjusted from 1.5316 to $1.5367 \mathrm{~m} \mathrm{~s}^{-1}$ such that equal drag was the starting point of the calculation), whereas in Fig. 3 the curve shows an intercept of approximately $-10 \%$. This indicates that in the case for equal power output in the 'free' and towing trial (i.e. $0 \% \Delta$ power


Fig. 4. Sensitivity analysis of equal power assumption: simulated relative difference in drag between the MAD- and VPM-method dependent on relative difference in power swimming with (towing trial) and without hydrodynamic buoy ('free' trial) using the VPM results as reference. Actual data points are given in the same graph.


Fig. 5. Sensitivity analysis of assumption that drag relates to speed squared: simulated relative difference in drag between the MAD- and VPM-method dependent on the value of the exponent of the power function: drag $=A v^{n}$.
'free' vs. towing trial) no equality of MAD-drag and VPM-drag is found, but actually an approximately $10 \%$ higher drag value for the VPM-method. Could this be due to the other assumption made in the VPM-method that drag relates to speed squared?

### 4.3. Sensitivity analysis of the exponent in the drag speed relationship diverging from 2

A relative small effect of the variation of the exponent on the relative difference in VPM-drag estimates is observed in Fig. 5. Previous studies show a range of exponents between 1.9 and 2.8 (Toussaint et al., 1988b) leading to errors of $15 \%$. The about $10 \%$ difference observed in Fig. 3 is found here for the exponent equal to 2 , while no difference in drag estimates occurs at an exponent value of 2.34 where the curve crosses the $x$-axis.

The previous calculations suggest that the equality of power output is not met in all tests that were performed. Another assumption that might be prone to violation is the equality of propelling efficiency in the 'free' and towing trial. Consider the hypothetical case that the drag of the buoy is nearing infinity. Hence, the speed of swimmer plus buoy approaches zero (tethered swimming). In the tethered swimming condition, no power will be used to overcome drag. All mechanical power will be expended giving a kinetic energy change to masses of water that are pushed back to generate propulsion. Thus, the part of the total mechanical power that is used beneficially to overcome drag might be reduced if the drag of the buoy relative the drag of the swimmer is too large. Further experiments with buoys of various drag characteristics might reveal the proneness to error of this equality of propelling efficiency assumption.

## 5. Conclusion

Although on average the two methods yield significant different active drag values, this does not imply that the two methods measure a dissimilar phenomenon. The observed differences can be explained by a violation of test assumptions. This could lead to the tentative conclusion that both methods measure essentially the same active drag phenomenon. Nevertheless, the question which physical phenomenon is assessed with the MAD-system and which with VPM is not finally resolved by the present time and demands further research.

## Acknowledgements

This research was supported by a grant from the National Olympic Committee of the Netherlands ( $\mathrm{NOC}^{*} \mathrm{NSF}$ ). Hans Elzerman and Fedor Hes are kindly acknowledged for providing facilities to pursue the research project. In the process of data collection, we received great assistance from Sander van der Meer, Martin Truijens, Mark de Niet, Maartje van Stralen, Eric Stevens, Hans de Koning and Ivo van der Hout. Joeri Beets is acknowledged for his assistance in the analysis of the data. Last but not least we like to thank the swimmers of TZA (Top-Zwemmen Amsterdam) for their participation in this study.

## Appendix. A

Listing of a Matlab program to simulate the effect of the violation of the equal power assumption in 'free' trial (swimming without the hydrodynamic buoy) vs. towing trial (swimming with the hydrodynamic buoy)
\% sensitivity analysis for the results of the velocity perturbation method (VPM)
$\%$ for the assumption that power to drag in the two swim trials (with and without buoy)
$\%$ is equal and that drag relates to speed squared \%
\% author Huub Toussaint
\% November 2002
\%
\% Matlab 5.1 for Apple Macintosh
clear
$\%$ drag $=A^{*} v^{\wedge} \mathrm{n} A$ and $n$ are the average of the 6 subjects in the present study see Table 2
$\mathrm{A}=21.32673846$;
$\mathrm{n}=2.336538462$;
\% drag of the hydrodynamic buoy is according to Kolmogorov: $\mathrm{Fb}=9.32 \mathrm{v}^{\wedge} 2$
$\mathrm{Ab}=9.32$;
\% power P to drag during the 'free' trial swimming without the buoy (average of 6 subjects; see Table 2) $\mathrm{P}=110.5237565$;
\% calculate the speed during the 'free' trial swimming arms only without buoy:
$\mathrm{v} 11=(\mathrm{P} / \mathrm{A})^{\wedge}(1 /(\mathrm{n}+1))$
$\%$ and the drag force on the basis of the MAD results will be:
$\mathrm{F}=\mathrm{A}^{*} \mathrm{v} 11^{\wedge} \mathrm{n}$;
\% calculate the speed (v21) of swimming with the buoy given the same power output to drag:
OPTIONS = foptions;
$\mathrm{v} 21=\mathrm{fmin}$ ('boeifunctie', $1,2, \mathrm{OPTIONS}, \mathrm{P}, \mathrm{A}, \mathrm{Ab}, \mathrm{n}$ )
\% What is the effect on calculated drag force if power in the towing trial (with hydrodynamic buoy
$\%$ differs from the power to drag swimming arms only:
$\%$ the power in the second trial is P 2 , which is between $60 \%$ and $102 \%$ of that in the 'free' trial
$\mathrm{P} 2=[0.6: 0.01: 1.02]$;
$\mathrm{P} 2=\mathrm{P} 2 .{ }^{*} \mathrm{P}$;
$\mathrm{k}=$ length $(\mathrm{P} 2)$;
$\%$ so power in the free trial is P resulting in a speed v11 and a drag of F based on MAD data \% now calculate the effect on drag according to VPM when power in the towing trial is not equal:
for $\mathrm{i}=1: \mathrm{k}$;
\% calculate the speed for the towing trial with the buoy
v22(i) = fmin('boeifunctie', 1,2, OPTIONS,P2(i),A,Ab,n);
$\%$ calculate the drag according to the equation of Kolmogorov and Duplisheva, (1992):
$\operatorname{Fdrag}(\mathrm{i})=\left(\mathrm{Ab}^{*} \mathrm{v} 22(\mathrm{i})^{\wedge} 3^{*} \mathrm{v} 11^{\wedge} 2\right) /\left(\mathrm{v} 11^{\wedge} 3-\right.$
v22(i)^3);
\% Calculate the difference in power between the 'free' and towing trial
deltaP(i) = P-P2(i);
$\%$ express difference in \% relative the power in the 'free' trial
deltaPprocent $(\mathrm{i})=100^{*}$ deltaP $(\mathrm{i}) / \mathrm{P}$;
\% calculate difference in drag force MAD-VPM deltaF(i) $=$ F-Fdrag(i);
$\%$ \% express difference in \% relative the drag estimated with MAD data in the 'free' trial deltaFprocent $(\mathrm{i})=100^{*} \operatorname{deltaF}(\mathrm{i}) / \mathrm{F}$;
end
$\%$ load data of actual experiment, i.e. column $\%$
$\Delta$ drag and column $\% \Delta \mathrm{P}$ of Table 2
loadfile $=$ ['deltaPdeltaF.txt'];
if exist $([$ loadfile $])=2$; eval (['load ', loadfile ,' -ascii’]);
end
figure(3);clf
plot(deltaPprocent,deltaFprocent)
hold on
xlabel(‘ $\Delta$ power 'free' trial vs. towing trial (\%)');yla-
bel(' $\Delta$ drag MAD vs. VPM (\%)');
title(['Sensitivity analysis of equal power assumption'])
$\operatorname{plot}($ deltaPdeltaF(:,2), deltaPdeltaF(:,1),'ro')
\% effect of assumption that drag relates to speed squared
$\%$ exponent n 1 between 1.9 and 2.7; The power is kept constant at P
$\mathrm{n} 1=[1.9: 0.01: 2.7]$;
$\mathrm{k}=$ length(n1);
$\operatorname{Fdrag}(\mathrm{i})=\left(\mathrm{Ab}{ }^{*} \mathrm{v} 22(\mathrm{i})^{\wedge} 3^{*} \mathrm{v} 11^{\wedge} 2\right) /\left(\mathrm{v} 11^{\wedge} 3-\mathrm{v} 22(\mathrm{i})^{\wedge} 3\right)$;
for $\mathrm{i}=1: \mathrm{k}$;
$\operatorname{Fdrag}(\mathrm{i})=\left(\mathrm{Ab}^{*} \mathrm{v} 21^{\wedge} 3^{*}\right.$ v11^n1(i) $) /$
(v11^(n1(i)+1)-v21^(n1(i)+1));
\% calculate difference in drag force MAD-VPM deltaF(i) = F-Fdrag(i);
$\%$ \% express difference in \% relative the drag estimated with MAD data in 'free' trial deltaFprocent $(\mathrm{i})=100 * \operatorname{deltaF}(\mathrm{i}) / \mathrm{F}$;
end
figure(4);clf
plot(n1,deltaFprocent)
xlabel('exponent of speed $v$ ');ylabel(' $\Delta \mathrm{drag}$ MAD vs. VPM (\%)');
title(['Sensitivity analysis of exponent speed $\left.=2^{\prime}\right]$ )
\% Simulation using the VPM method as reference: use data of subject ' S '
$\mathrm{v} 1 \mathrm{t}=1.7224 ; \%$ average speed in 'free' trial of subject S
$v 2 t=1.5316 ; \%$ average speed in the towing trial
$\mathrm{At}=20.746 ; \mathrm{nt}=2.177 ; \% \mathrm{MAD}-\mathrm{drag} \mathrm{At}^{*}{ }^{\wedge}{ }^{\wedge} \mathrm{nt}$
Fvpmt $=$ Ab.* v2t.^3.*v1t.^2./(v1t.^3-v2t.^3); \%drag according to VPM
Fmadt $=$ At. ${ }^{*}$ vlt. ${ }^{\wedge}$ nt; \%drag according to MAD.
Avpmt $=A b .^{*} v 2 t .{ }^{\wedge} 3 . /\left(v 1 t .{ }^{\wedge} 3-v 2 t .{ }^{\wedge} 3\right)$;
Pvpmt $=$ Avpmt* ${ }^{*} 1 t^{\wedge} 3 ; \%$ power to drag according to VPM
c2 $=$ [0.85:0.01:1.02]; \%create range of speeds in towing trial deviating from $1.5316 \mathrm{~m} \mathrm{~s}^{-1}$
$\mathrm{v} 2 \mathrm{te}=\mathrm{c} 2 .{ }^{*} \mathrm{v} 2 \mathrm{t}$;
$\mathrm{k}=$ length(n1);\%number of speed point
for $\mathrm{i}=1: \mathrm{k}$;
$\operatorname{Pvpm} 2 \mathrm{t}(\mathrm{i})=$ Avpmt $^{*}$ v2te(i)^ $3+\mathrm{Ab}^{*}$ v2te(i) ${ }^{\wedge} 3 ; \%$ calculate power in towing trial deltaPt2(i) $=$ Pvpmt-Pvpm2t(i); \%determine power difference from 'free' trial deltaPprocentt2(i) $=100^{*}$ deltaPt2(i) $/$ Pvpmt; $\%$ express difference in $\%$ relative the power in 'free' trial
Fdragt2(i) $=\left(\mathrm{Ab}^{*} \mathrm{v} 2 \mathrm{te}(\mathrm{i})^{\wedge} 3^{*} \mathrm{v} 1 \mathrm{t}^{\wedge} 2\right) /\left(\mathrm{v} 1 \mathrm{t}^{\wedge} 3-\right.$
v2te(i)^3); \% calculate drag VPM given speed in the towing trial
deltaFt2(i) $=$ Fmadt-Fdragt2(i); \% calculate difference in drag force MAD-VPM $\%$ express difference in $\%$ relative the drag estimated with MAD data in 'free' trial deltaFprocentt2(i) $=100^{*}$ deltaFt2(i)/Fmadt;
end
figure(5);clf
plot(deltaPprocentt2,deltaFprocentt2)
hold on
xlabel(' $\Delta$ power 'free' vs towing trial (\%)');ylabel ('drag MAD vs VPM (\%)’);
title(['Sensitivity analysis of equal power assumption'])
$\operatorname{plot}(\operatorname{deltaPdeltaF}(:, 1), \operatorname{deltaPdeltaF}(:, 2)$, 'ro')
function $y=$ boeifunctie $(\mathrm{v} 21, \mathrm{P}, \mathrm{A}, \mathrm{Ab}, \mathrm{n})$
$\mathrm{y}=\operatorname{abs}\left(\mathrm{P}-\left(\mathrm{A}^{*} \mathrm{v} 21^{\wedge}(\mathrm{n}+1)+\mathrm{Ab}{ }^{*} \mathrm{v} 21^{\wedge} 3\right)\right)$;

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